double factorial

Def

= , if and is integer.

= , if

undefined , otherwise.

Example

= 1

= 1

= =

= =

= =

= =

Property

1. = for all n is positive integer.

Proof:

Method 1:

By definition, we consider two cases:

Case 1:

When n is even

=

=

=

Case 2:

When n is odd

=

=

=

Method 2:

Proof by induction,

Base step:

1. When n is equal to 1,

= = =

Inductive step:

1. When n is equal to k,

By inductive hypothesis,

=

We have to prove that

=

=

=>

=

=

=

Hence proved.

2. =

Proof:

=

=

=

=

=

=

3. =

where

is permuation of n from 2n different objects.

Proof:

=

=

=

=

=

=

=

=

3. = for is a positive integer

4. =, if is odd.

5. = , if is a complex number.

6.

=

= , if is odd

, if n is even.

7.

=

=

Asymptotics

The asymptotics of double factorial is:

~ , if is even

~, if is odd

Approximation

~

Ref

[Double factorial - Wikipedia](https://en.wikipedia.org/wiki/Double_factorial)